SPIRAL FRACTAL ARRAYS

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ABSTRACT

This paper describes the use of fractal arrangements for the design of planar antenna arrays. Spiral initiators were used to play the chaos game, which generated our fractal array. Once these antenna arrays were produced, the resulting radiated field was calculated using scripts written in Matlab. Variations on the number of arms and inner circles in our spiral initiators were tested to characterize which formation would allow for our fractal array to perform equivalently or better than a set of random elements placed on a given circular area. The compared random array held the same number of elements as our spiral array did which was usually 441. The resultant spiral array behaved much like tapered arrays in that the concentration of points tended towards the center. The overall behavior of the fractal array in comparison to the random array was comparable in terms of directivity and average sidelobe level. Additional attempts to make the spiral array less tapered allowed for possible characterization as a “multi-fractal” array by changing the number of points that made up inner circles of the initiator spiral. These results also proved to be comparable to random and spiral arrays. Implications of these results and further exploration needed, is discussed.

1. INTRODUCTION/HISTORY

Antenna arrays are systems that need to be improved upon to be able to produce the ideal radiation propagation with a minimized number of points held within a smaller area. Typical arrays involve elements placed on an x-y plane where a number of antenna elements are placed either scattered in a random fashion or in some ordered way. The radiated fields that are propagated from these various arrays are quite different, each has its own distinguishable properties that are ideal for certain uses.

Antennas traditionally used for scanning purposes, which are the main focus in our research are evaluated by two of their main structures: the clarity of a “point-like” mainbeam, and a low sidelobe level (discussed below). Ordered arrays have a better quality of mainbeam, but as a drawback, the sidelobes are very high. Aside from the failure in terms of sidelobes, ordered arrays are also extremely dependant on the correct placement of antenna elements. In comparison, random arrays are much more robust with a much lower sidelobe level, but the drawback remains of a warped mainbeam.

Since the 1980’s, fractals have been investigated in physics and engineering for their apparent combination of both ordered and random configurations to give the resultant fractal structures. This research explores fractal configurations in an attempt to combine the attributes of ordered and random antenna arrays.

We hope to present a clearer understanding of fractals, antenna arrays, and how fractals play a part in antenna engineering. Our project aims to explore the properties of fractal antenna arrays and be able to characterize the advantages that they give as opposed to random and ordered arrays. Our long-term goal is to find an ideal fractal array that performs better than any random or ordered array, we also hope to classify which fractal attributes will lead to a better array.

2. PROBLEM STATEMENT AND SOLUTIONS

2.1 Antenna Arrays

2.1.1 Linear Arrays

In constructing linear and planar arrays, the radiation properties of distinct patterns must be analyzed in order to optimize the array for certain uses. We first looked at linear antenna arrays upon which we placed “n” number of elements. We used Matlab to calculate the constructive and destructive interference in terms of the array factor (AF), that characterizes the radiated field.

$$AF = \sum_{n=0}^{N-1} I_o e^{-i2\pi N\frac{d}{\lambda}\cos\theta + \alpha} \qquad (1)$$

Here, N is the number of elements in the X-axis, I_o is the charge per element, d/lambda is the ratio of the distance between each element in wavelengths, and alpha is a phase shift which changes the direction of the mainbeam. To simplify calculations, we kept the charge equal for all elements. We also chose to keep alpha equal to zero due to the effects that phase has on the mainbeam. Matlab used a “while” loop function to calculate the sum of the array factor for n elements a distance apart with uniform voltage. As the elements were placed within 1 wavelength, there was constructive interference such that there was always one main lobe with smaller sidelobes as more elements were added. Figure 1 shows the mainbeam with high sidelobes for both a linear and planar array. These structures were normalized by dividing the AF by the number of points, and then by taking the $20*\log_{10}$ of the array factor. This scaling allowed for better analysis of the sidelobes and their characteristics. Our plots had a threshold at –30dB for better graphical analysis.

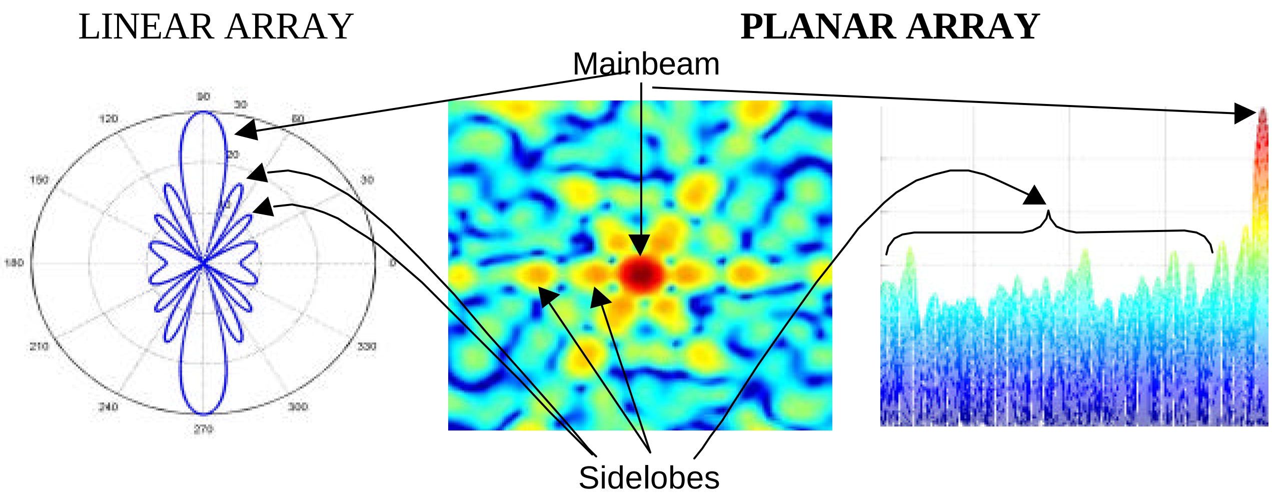


Figure 1: Main lobe with high side beams.

Two-dimensional antenna arrays allow for more flexibility and variety in placements of array elements. As discussed earlier, two main ways in which elements have been placed on planar arrays are to place them on a grid or to randomly scattering them about a certain area. Although both methods result in sidelobes, both have their advantages. Once again, Matlab used a while loop function to calculate the sum for the array factor.

$$AF = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} e^{-i2\pi M(\frac{d}{\lambda})\cos\theta\sin\phi}\, e^{-i2\pi N(\frac{d}{\lambda})\cos\theta\cos\phi} \qquad (2)$$

To simplify operations we let $f_x = \frac{x}{\lambda r}$ and $f_y = \frac{y}{\lambda r}$ so that the array factor becomes:

$$AF = \sum_{n=0}^{N-1} e^{-i2\pi(f_x x + f_y y)} \qquad (3)$$

Figure 2 illustrates the problem geometry where we examine the array factor projected on the Z-axis. Here, R is the observer's distance from the origin. Matlab also used "while" loops to generate the points on a 2x2 matrix, which was then calculated to plot the array factor on a three-dimensional plot.

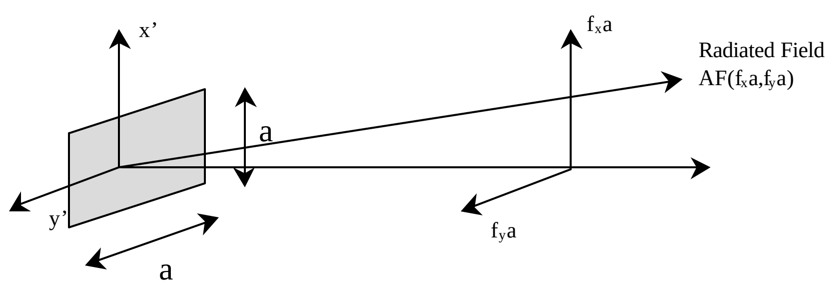


Figure 2: The geometry of the problem, AF is measured in the far-zone.

2.1.3 Characterization of Optimized Arrays

An optimized antenna for our purposes of scanning would have no sidelobes and a clear point-like mainbeam. The benefit of having only one beam is apparent when we consider uses of theses antenna in airports. As seen in Figure 3A, if the same array in Figure 1 were used in an airport, the sidelobes would easily cause air traffic control to confuse a large airplane at the height of the sidelobes with a small plane at the peak of the mainbeam. Another characteristic of an optimized beam involves a thin single mainbeam. Figure 3B shows the catastrophe that could happen if a thick mainbeam were to confuse two airplanes as one large plane. Although these examples may seem a bit extreme, our main goal is to minimize any and all interference in order to allow for complete assurance that what is being observed by our antenna is what is in fact there.

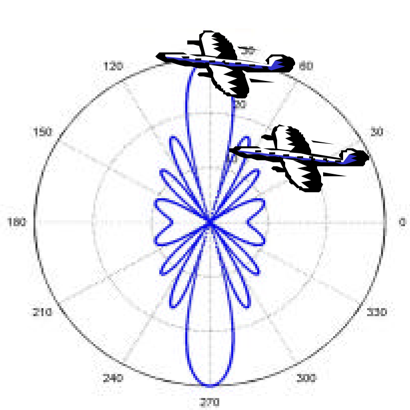


Figure 3A: Array Factor with high side

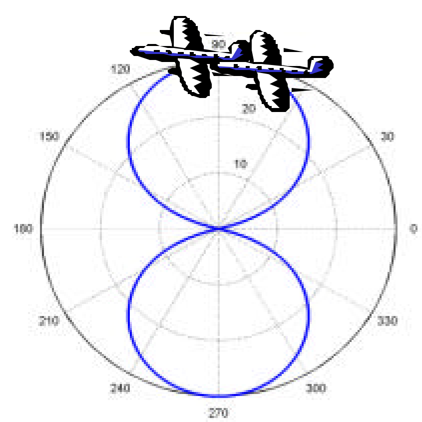


Figure 3B: Array Factor with thick mainbeam.

2.2 Fractals

2.2.1 Fractal Basics

Although fractals are mainly discussed in mathematical forums, they exist in all parts of nature. For example Mandelbrot [1] discusses the basics of fractal theory as applied to the characteristics of a coastline (see Figure 4). The length of a coastline depends on the size of the measuring yardstick. As the yardstick we use to measure every turn and detail decreases in length, the coastline perimeter increases exponentially. As the view of a coastline is brought closer, we discover that within the coastline there lie

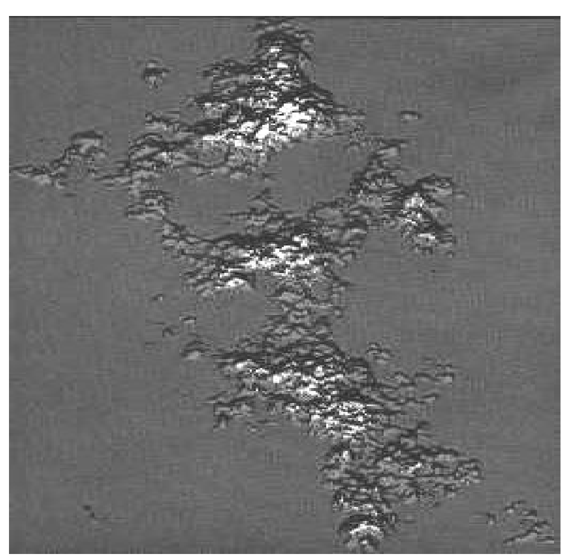


Figure 4: Fractal-generated coastline.

miniature bays and peninsulas. As we examine the coastline on a rescaled map, we discover that each of the bays and peninsulas contain sub-bays and sub-peninsulas. There is a self-similar trait observed as we look at the coastline at various resolutions. The number of microscopic structures begin to approach infinity. In fact, because of the immense number of irregularities, the physical length of a coastline is virtually infinite.

Self-similarity (seen in the coast example above) is defined by structures that look the same at variable magnifications. This recurring self-similarity is one of the many attributes of many fractals. Much like the coastline described above, any small part in a self-similar fractal is going to look exactly like the fractal as a whole. Further illustration can be seen in Figure 5, which shows various stages of the well-known Sierpinski gasket.

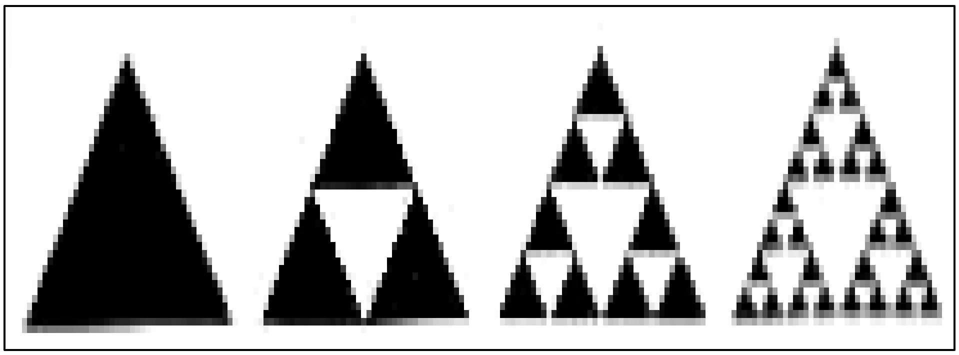


Figure 5: Sierpinski gasket.

The Sierpinski gasket is formed by the same inverted triangle redrawn multiple times at different scales. Every time a new "empty" triangle is formed, the next step of growth places the inverted triangle on all available spots, therefore creating more triangles to fill in the stage of growth.

Another type of figure with a slightly different system of construction uses a generator/initiator relationship. This construction begins by placing an "initiator," which will be the base format for the figure. The initiator is then divided into a collection of lines upon which the generator(s) will be placed. Figure 6 shows the initiator and its first stage of growth where the lines are replaced, or added to, by one of the two generators.

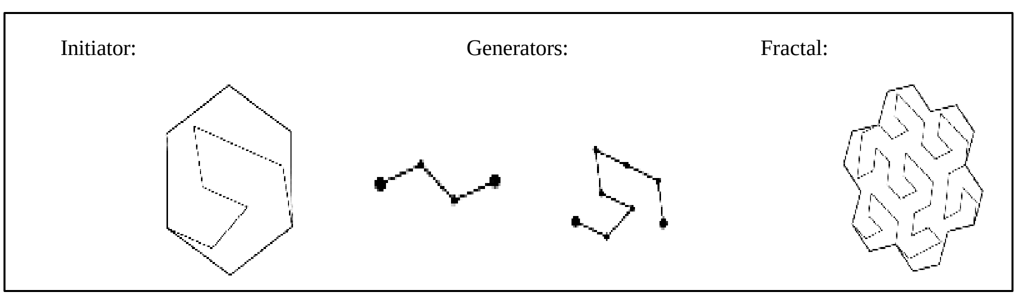


Figure 6: Initiator/generator fractal.

2.2.2 Chaos game and mapping

The process of making fractals by an initiator and a generator can be simplified by choosing the same Figure for both. This “mapping” quality of a Figure upon itself can be done using the chaos game. To clarify the chaos game, we will give an example of generating the same Sierpinski gasket described above using the random qualities of the chaos game.

We begin the chaos game, by choosingse our initiator shape as a triangle. We then “create” a multi-faced die, that has the same number of sides as the number of points we have on our initiator shape. In this case, our die is going to have three sides, as our triangle has three sides. Our next step involves choosing an initial random point anywhere in or around the shape. Now we roll our die. For whichever side it falls on we place a new point that is half the distance between our initial point and the chosen vertex. This action gives us a new initial point from which we will take the next "half distance". After repeating each of these steps, the resultant image is the one seen below in Figure 8.

Although there is a large random factor that takes place in playing the chaos game, the resultant set of points seems quite ordered. For these reasons, we chose to create our fractals by choosing a variety of Figures and playing the chaos game with them as initiators. To keep our points within the actual fractal structure, we chose our initial points at the beginning of the game to be the various vertices of our initiator shape. We also chose to run the game as many iterations as wanted, and then restarted at the next vertex in order to maintain a more even distribution of points. Finally, we varied the "mapping factor" which is the amount by which points are taken. In the Sierpinski gasket, we took half the distance between points which meant that we were actually shrinking our triangle by one half. For our other fractals we used mapping factors of 1/10 in order to avoid too much overlap of the reduced shapes.

3. METHODS AND RESULTS

3.1 Circular Disc Arrays

Antenna theory states that any preferred side on the array causes for sidelobes along a line perpendicular to the side on the array. For this reason we want to minimize, or eliminate, all preferred straight sides on the array. One way to do this is to create a circular array, which in effect has an infinite number of sides, therefore eliminating any and all perpendicular sidelobe lines. An ideal antenna array would be a solid disc which has all points evenly distributed. The drawback of keeping this array for regular use is that too much power is needed. The number of points in the array is not necessarily infinite, but making this array into a smaller number of discrete elements would make the system much more efficient and inexpensive. An example of a solid disc made up of 6,736 discrete elements is seen below in Figure 9. Figure 10 shows two different views of the array factor of the array in Figure 9.

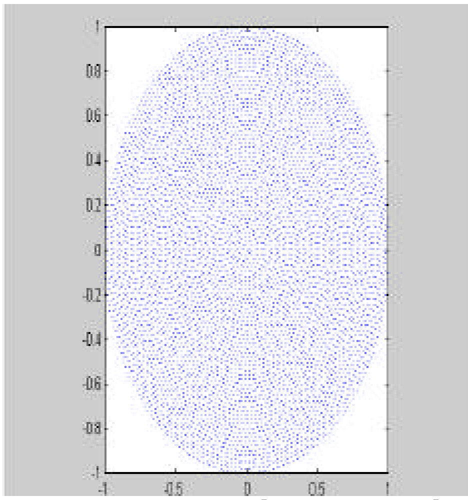
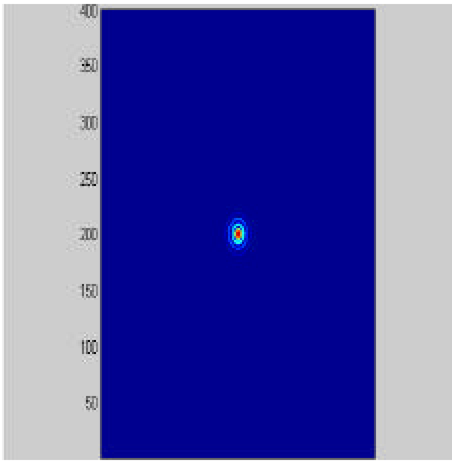


Figure 9: Discrete disc array made up of 6736 points



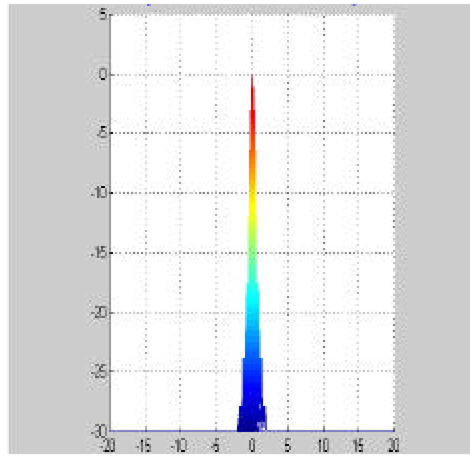


Figure 10A: Array factor seen from above
Figure 10B: Side view of array factor

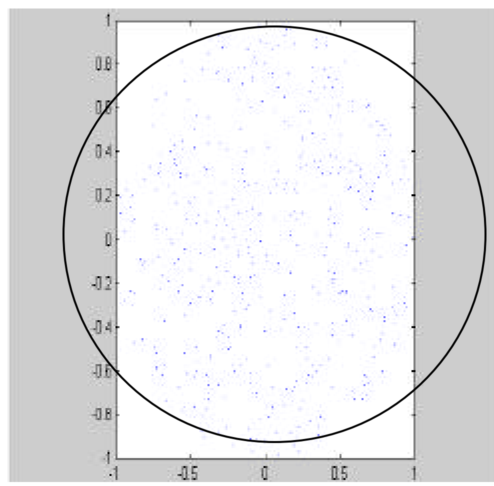
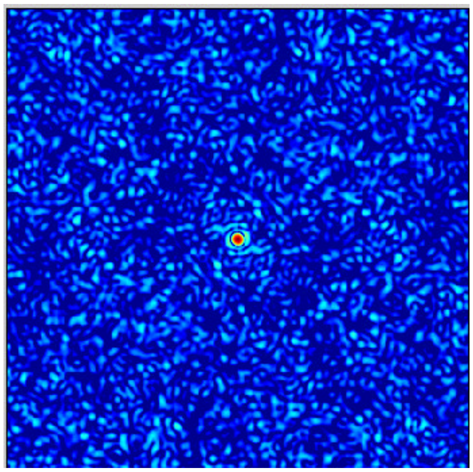


Figure 11: Random array of points in a circular area consisting of 441 points

3.2 Random Disc Arrays

One way of minimizing the number of points in a circular area is to place points randomly within a given circular area. Last year's study of random and ordered arrays proved that points that were placed in random arrangements in a given rectangular area performed well in terms of keeping a low sidelobe level, but produced a warped mainbeam. The mainbeam varied from being rectangular to being a collection of two or three high peaks that joined together to produced a mainbeam. This year, we placed random points in a circular area, and results were similar, but there was a better quality in the "roundness" of the mainbeam. Our main reason for testing these random arrays was to have a comparison to our fractal arrays. In terms of sidelobe levels, there is a range of antenna elements used where fractal arrays perform better than random arrays, and another higher range of points where random arrays perform better than an fractal structure. In comparing our fractal arrays to random arrays, we hope to find that "crossover" of performance.



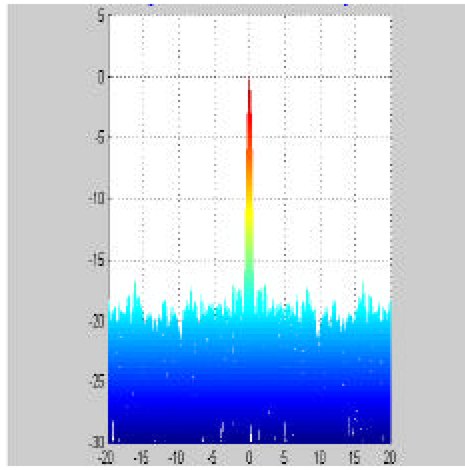


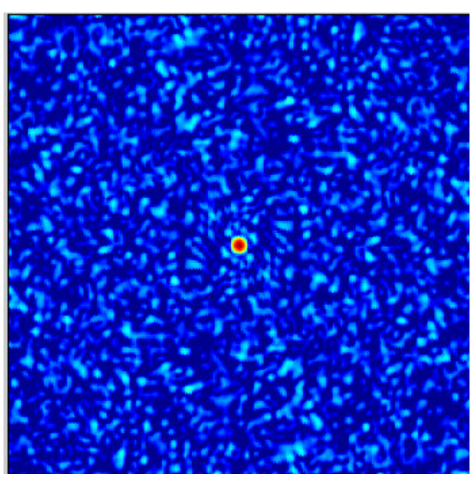
Figure 12A: Array factor seen from above
Figure 12B: Side view of array factor

3.3 Spiral Arrays

In attempts to produce a circular array we decided to analyze spiral arrays. To construct these spiral initiators, we wrote a Matlab script that plotted circles around the origin with varying radii. As these circles were made up of discrete elements, the collection of circles within another produced types of arms that produced preferred sides. To eliminate this, we gave a different initial phase shift to each of the inner circles to produce a spiral form. This construction can be seen in Figure 11 below.

[Figure 11a,b,c]

Once we constructed these spirals, the chaos game was played a total of 300-500 times. As mentioned earlier, this total number means that the chaos game was restarted every 30 times in a shape that had 10 initial points. We tested various distributions of the radii of the inner circles, as well as the number of arms that were seen within the spiral structure. These arrays performed very well. We were able to attain a really low sidelobe level as well as a well defined mainbeam (Figure 12).



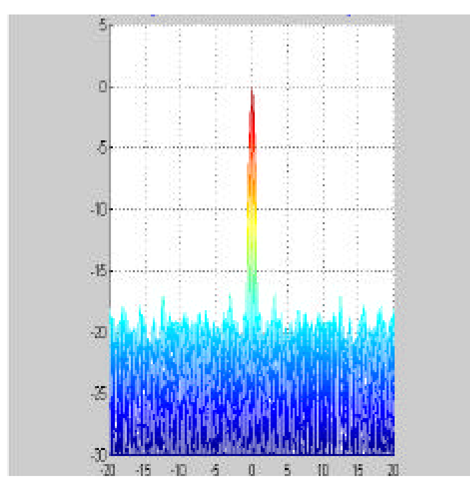


Figure 12A: Array factor seen from above
Figure 12B: Side view of array factor

4. DISCUSSION OF RESULTS (RANDOM VS. SPIRAL)

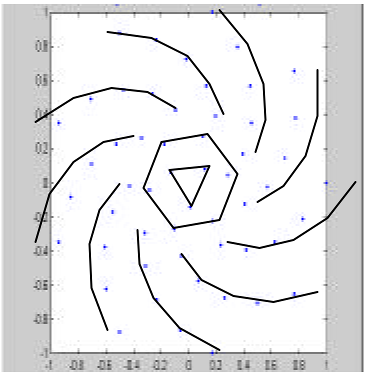
In comparing testing our spiral arrays (9 arms, 7 inner circles, with repeated sets of iterations every 6 times for a total of 441 points) with random circular arrays of 441 points, we found that the average sidelobe level for both arrays was about –17dB. Their overall performance was very similar. There was not a well defined point that where we could tell that the fractal array performed better than a random one. The main reason for this is because of the tapered characteristics of our original spiral array.

In terms of mainbeams, both also performed quite similarly. The only difference that the spiral arrays had from the random arrays was a thicker mainbeam. In measuring directivity, they both came within a 3-5% difference, with a lower directivity for the spiral arrays. The following results show a trace of directivity measurements and they're level off at higher visible ranges.

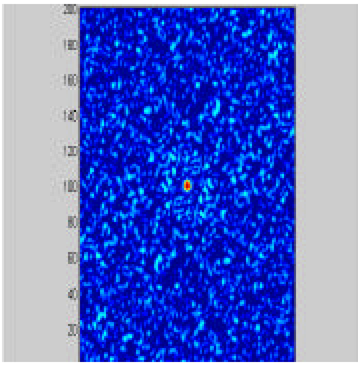
(viewable distance from center)	**Spiral Array**	**Random Array**
45 units	424.1070	439.3319
47 units	423.7061	442.0472
59 units	438.2889	441.1471
69 units	437.7500	448.7248
71 units	440.3611	445.5012

5. RECOMMENDATIONS

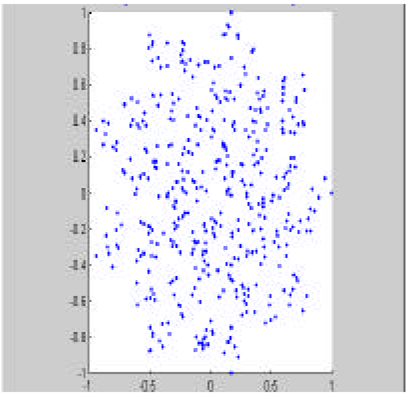
In attempts to make the array less tapered, further research will involve exploring multifractal scalings by changing the initiator shape by pulling out points, and maybe trying additional circular fractals. In the last two days of research we tested new fractal spirals that had initiator points pulled out from the center, and they performed at the same level of the arrays previously mentioned, although with slightly fewer points. This is evidence that it may be possible to recreate the results attained in this research with arrays that have a fewer number of elements using this new organization, which would prove the system to be more efficient and less costly.



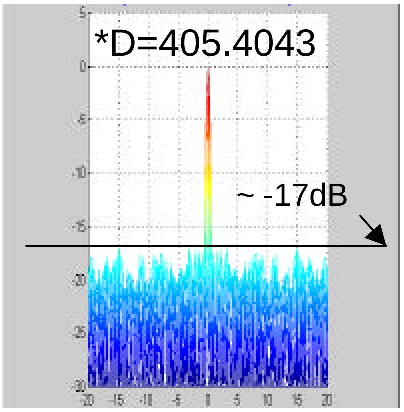
Spiral initiator array with inner points removed.



Array factor of new fractal array



New spiral fractal array after playing chaos game



Side view of array factor: equivalent behavior to random and fractal arrays using 423 elements

More work in examining the effect of tapered arrays must be pursued. Aside from using a different initiator from which to play the chaos game, we may also simply ignore the last set of 20 or 30 elements since the last sets of points originated from starting the chaos game iterations at the centermost points.

6. ACKNOWLEDGMENTS

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