The Geometry of Nature

- The Euclidean Perspective
  - Circles, rectangles, triangles
  - Spheres, cubes, cylinders, cones

- The Non-Euclidean Geometry of Nature
  - Clouds are not spheres
  - Mountains are not cones
  - A coastline is not a curve
  - Lightning is not composed of straight lines
What is a Fractal?

- Self-similarity
- Dimension
- Lacunarity

By Frederick Diaz
## Cantor Set

<table>
<thead>
<tr>
<th>Stage</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>![Stage 0 Diagram]</td>
</tr>
<tr>
<td>1</td>
<td>![Stage 1 Diagram]</td>
</tr>
<tr>
<td>2</td>
<td>![Stage 2 Diagram]</td>
</tr>
<tr>
<td>3</td>
<td>![Stage 3 Diagram]</td>
</tr>
<tr>
<td>4</td>
<td>![Stage 4 Diagram]</td>
</tr>
<tr>
<td>5</td>
<td>![Stage 5 Diagram]</td>
</tr>
</tbody>
</table>

The dimension of the Cantor Set can be calculated as:

\[
D = \frac{\ln(N(\varepsilon))}{\ln(1/\varepsilon)} = \frac{\ln(2)}{\ln(3)} = 0.63093
\]
Antennas and Arrays

- Single Antenna ➔ Broad Radiation Pattern
  - Television networks
  - Radio stations

- Array ➔ Point-to-Point and Preferred Coverage
  - Flight towers for pinpointing location of planes
  - Medical imaging and scanning

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Array Factor

Array Factor: Mainbeam, Sidelobes

Problem Geometry

\[ \sin(\theta) \approx \text{to normal to array} \]
Traditional Array Configurations

- Along a Periodic Lattice
  - 200 Elements (20x20)
- Random within a Given Area
  - 200 Elements

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Fractal Arrays

- Random Fractals => Low Sidelobes + Robustness
- Combine order and disorder to restrict randomness (tethering)
Creating Cantor Arrays

Stage 0

Stage 1

Stage 2

Stage 3

Stage 4

Stage 5

$$D = \frac{\ln(N(\varepsilon))}{\ln(1/\varepsilon)} = \frac{\ln(2)}{\ln(3)} = 0.63093$$

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Motivation and Reference Points

Base Case: Cantor Ring Array (N = 556)
3-gap Cantor Bar, Stage 2, D = 9/10

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Goals

- Design discrete Cantor ring arrays comparable to continuous case
- Design arrays superior to or comparable to periodic and random cases with regard to:
  - Number of elements
  - Sidelobe level
  - Visible range
  - Robustness
  - Mainbeam quality
Azimuthal Spacing:
- Periodic
- Random

Azimuthal Spacing:
- Tethered
- (3/4)2π/n
- Taper Out
- Inverse Taper

Azimuthal Element Configurations
Azimuthal Spacing:
 Periodic
\(N = 556\)

Azimuthal Spacing:
Tethered (Range \((3/4)2\pi/n\))
\(N = 556\)

-3 dB

Lowered sidelobe power by a factor of 2 when number of elements kept constant.
Achieved comparable performance using half as many elements.
Next Steps

Other Element Configurations

- Tapered
- Inverse-tapered
- Fractal in azimuthal direction