Visualization of Reachability Graphs in Hybrid Systems

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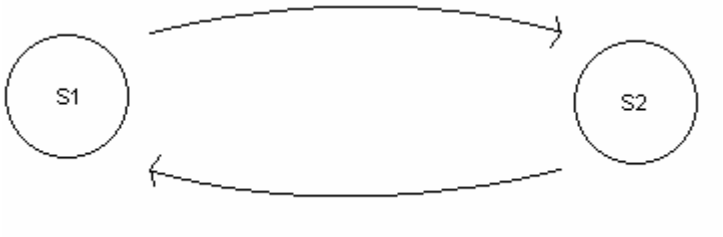
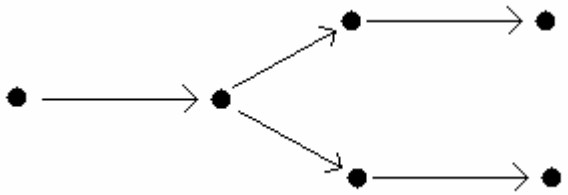
A couple of definitions

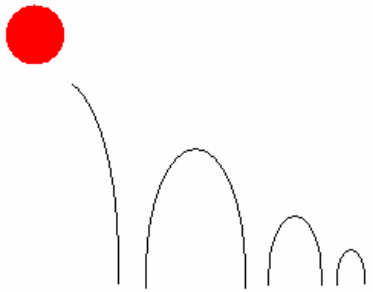
- Finite State Machine
- Directed Graph





What are “hybrid” systems?

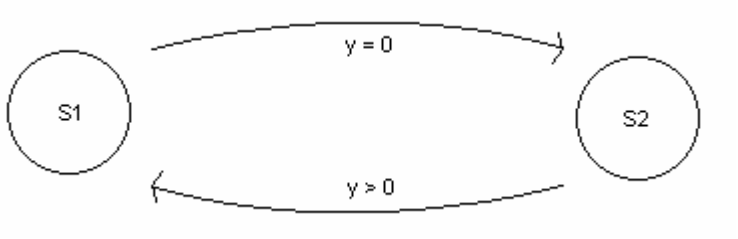
- Some systems cannot be modeled by discrete/continuous equations alone
- Example: a bouncing ball



- The hybrid systems approach combines discrete equations with continuous ones to more accurately model systems.

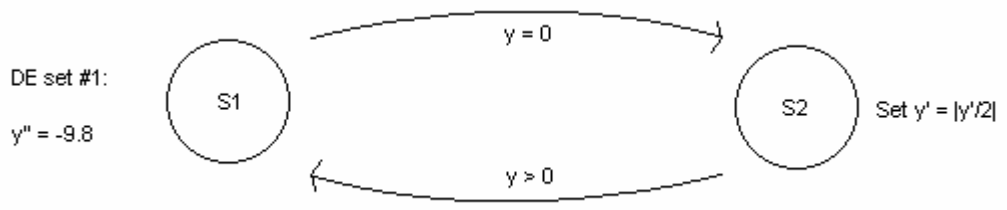
Hybrid Systems Defined

- Mathematically, a hybrid system is defined by sets of variables, continuous equations, discrete states and discrete transitions.
- The discrete transitions can be thought of as a finite state machine, with the input being one or more variables.



Hybrid Systems Defined 2

- The continuous component is one or more sets of differential equations.
- Whenever an “invariant” (usually based on variable values) stops being satisfied, the FSM makes a transition to a new state.
- Once in the new state, we can switch continuous equations, set or reset variables, or anything else we may want to do.



Hybrid System Analysis

- The N variables of the system form the basis for an N-dimensional vector space.
- This state space is partitioned into convenient sections (in our case N dimensional rectangles, or N-tangles).

Hybrid System Analysis 2

- Now that we have all of these partitions set, up we make a directed graph.
- We use each N-tangle as a node.
- With the combined knowledge of all the DEs we have from the model, we can interpret them as N-dimensional vector fields.
- The faces of the partitions are just surfaces in N-space.
- For each node, we determine whether there is flux through its faces. If so, we make a directed edge from the “outflux” node to the “influx” node.

Reachability

- We now have a directed graph where each node has a unique N-tangle associated with it.
- We generally want to be able to ask “From this starting node, is it possible to reach this other node in the graph?” or questions of that sort.
- This is called “reachability analysis”.

Visualization

- Another analysis problem is how to visualize an N-dimensional directed graph.
- The graph is most easily visualized by making the N-tangles appear as normal rectangles. We can do this by projecting the rectangles into a 2D plane determined by selecting 2 variables.
- By seeing multiple plots at the same time, you can get a large amount of information about the graph.

Our application: BIOCharon

- BIOCharon is a package designed to simulate biological models using the hybrid systems method.
- We run the hybrid systems analysis on an SBML (Systems Biology Markup Language) model and analyze the outputted graph.
- Variables are substance concentrations and other relevant bio data.

Software Demo: simple 3 variable system